Advanced Econometrics II
Homework Assignment No. 3

Deadline: 26.01.2015, 23:59

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Please submit your (typed) solution in a `pdf` file. Please motivate all your answers. For programming exercises (if there are any), the code has to be put, together with the main `pdf` solution file, in an archive file (e.g. `zip` or `rar`). Each code file shall contain your name, either as a comment or in its name.

Question 1

Consider the linear regression model

$$y = X\beta + u, \qquad \mathbb{E}[u_t|X_t] \neq 0, \quad \mathbb{E}[u_t|W_t] = 0, \quad \mathbb{E}[uu^T] = \Omega,$$

where y is $n \times 1$, X is $n \times k$ and W is a $n \times l$ matrix containing valid and relevant instruments, with $l > k$. Suppose that Ω is **known** and let Λ be a $l \times l$ weighting matrix that is PD[1] and nonrandom.

(a) *(GMM criterion function)*
Give the formula for the GMM criterion function for a specific choice of Λ. Use this formula to derive the (in general inefficient) GMM estimator for β. Show that the obtained estimator is the same as the one delivered by solving the moment conditions

$$J^T W^T (y - X\beta) = 0$$

for the $l \times k$ matrix J when $\Lambda = JJ^T$. Assuming that the above solution is unique, show that the matrix JJ^T is of rank k, which implies that it is PSD but not PD.

(b) *(Asymptotic distribution)*
Now, consider the weighting matrix of the form $\Lambda = \mathbb{I}_l$. Derive the asymptotic distribution of the resulting GMM estimator.

(c) *(Efficiency)*
Show that the above estimator is inefficient so that with a different weighting matrix Λ^* you can obtain a more efficient estimator. Suggest a form for Λ^* which leads to an asymptotically efficient estimator (and prove it indeed does).

[1] As before, P(S)D stands for positive (semi)definite.

Question 2

Consider the setup from the previous question, this time, however, assume that the covariance matrix of the error terms Ω is **unknown**.

(a) *(Heteroskedasticity of unknown form)*
Describe how to obtain an efficient, **feasible** estimator of β in the case of Ω being **diagonal**. Why is this estimator asymptotically equivalent to the efficient GMM estimator?

(b) *(Sample autocovariance matrices)*
Consider the **sample autocovariance matrix** of order j given by

$$\hat{\Gamma}(j) = \frac{1}{n} \sum_{t=j+1}^{n} \hat{u}_t \hat{u}_{t-j} W_t^T W_{t-j},$$

where $\hat{u}_t$ stands for a typical residual. Argue that $\hat{\Gamma}(j)$ is not a consistent estimator of the true autocovariance matrix for arbitrary j. Find the form of the $n \times n$ matrix $U(j)$ such that

$$\hat{\Gamma}(j) = \frac{1}{n} W^T U(j) W.$$

(c) *(Heteroskedasticity and/or autocorrelation of unknown form)*
Next, we want to drop the assumption that Ω is diagonal. How would you proceed to obtain the feasible and efficient estimator of β now? Give the formula for the corresponding, feasible criterion function Q_{FGMM}.

(d) *(Testing linear restrictions)*
Rewrite the basic model as

$$y = X_1\beta_1 + X_2\beta_2 + u,$$

where X_1 is $n \times k_1$ and X_2 is $n \times k_2$, with $k_1 + k_2 = k$. Propose a test statistic based on Q_{FGMM} to test the linear restrictions $\beta_2 = 0$.